## Precalc Warm Up # 4 -1

Turn in last week's warm up.

Definition of a circle:
The set of all points equidistant from a fixed point (the center).

1. (x,y) lies on a circle with center (h,k) with radius r. Derive the equation of the circle. Hence, find the equation of the circle with

a) center (-2,4) & radius 3 b) point (2,8) & center (-3,5)

2. Find the midpoint of (-2,3) and (4,10)

3. Find a relationship between x and y so that (x,y) is equidistant from (2,-6) and (-10, 2)

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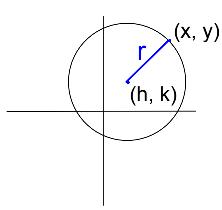
1. (x,y) lies on a circle with center (h,k) with radius r.

Derive the equation of the circle.




* Start with distance formula!

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

$$(x-h)^2+(y-k)^2=r^2$$

Find the equation of the circle with:

a) center (-2,4) & radius 3 b) point (2,8) & center (-3,5)

$$(x+2)^2+(y-4)^2=9$$

$$(x+3)^2+(y-5)^2=r^2$$

$$(2+3)^2+(8-5)^2=r^2$$

$$25+9=r^2$$

$$34=r^2$$

$$(x+3)^2+(y-5)^2=34$$

2. Find the midpoint of (-2,3) and (4, 10)

$mp = \left(\frac{-2+4}{2}, \frac{3+10}{2}\right)$

$= \left(1, \frac{13}{2}\right)$

the equation of the perpendicular bisector.

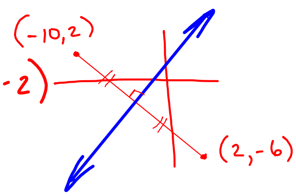
3. Find a relationship between x and y so that (x,y) is equidistant from (2,-6) and (-10, 2)

$y - y_1 = m(x - x_1)$

midpt $(x_1, y_1) = (-4, -2)$

$\left(\frac{-10+2}{2}, \frac{2-6}{2}\right)$

$y + 2 = \frac{3}{2}(x+4)$




$m = \frac{2+6}{-10-2} = \frac{8}{-12}$

$= -\frac{2}{3}$

$m\perp = \frac{3}{2}$

HW Questions?

In Exercises 13–24, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

15. (−4, 10), (4, −5)

In Exercises 29 and 30, find x so that the distance between the points is 13.

29. (1, 2), (x, −10)

30. (−8, 0), (x, 5)

$\sqrt{(x-1)^2 + (-10-2)^2} = 13$

$x^2 - 2x + 145 = 169$

$x^2 - 2x - \quad = 0$

$\boxed{x = 4, -20}$

$\sqrt{(x+8)^2 + (5-0)^2} = 13$

$(x+8)^2 + 25 = 169$

$(x+8)^2 = 144$

$x + 8 = \pm 12$

$x =$

In Exercises 31 and 32, find $y$ so that the distance between the points is 17.

**31.** (0, 0), (8, $y$)

**32.** (−8, 4), (7, $y$)

$\boxed{y = 12, -4}$

In Exercises 33 and 34, find a relationship between x and $y$ so that $(x, y)$ is equidistant from the two given points.

Looking for the equation of the ⊥ bisector of the segment.

**33.** (4, −1), (−2, 3)

34. $(3, \frac{5}{2})$ (-7, -1)

Midpt: $\left(\frac{4-2}{2}, \frac{-1+3}{2}\right)$

$(1,1)$

$m = \frac{-1-3}{4+2} = -\frac{4}{6} = -\frac{2}{3}$

$m\perp = \frac{3}{2}$

$\boxed{y - 1 = \frac{3}{2}(x-1)}$

In Exercises 31 and 32, find $y$ so that the distance between the points is 17.

**31.** (0, 0), (8, $y$)

**32.** (−8, 4), (7, $y$)

In Exercises 33 and 34, find a relationship between x and $y$ so that $(x, y)$ is equidistant from the two given points.

$mp = \left(\frac{3-7}{2}, \frac{1}{2}\left(\frac{5}{2} - \frac{2}{2}\right)\right)$

**33.** (4, −1), (−2, 3)

34 $(3, \frac{5}{2})$ (-7, -1)

$mp = \left(-2, \frac{3}{4}\right)$

$\boxed{y - \frac{3}{4} = -\frac{20}{7}(x+2)}$

$m = \dfrac{\frac{5}{2}+1}{3+7}$

$= \dfrac{\frac{7}{2}}{10}$

$\frac{7}{2} \cdot \frac{1}{10} = \frac{7}{20}$

In Exercises 35–42, determine the quadrant(s) in which $(x, y)$ is located so that the given conditions are satisfied.

**38.** $x < 0$ and $y > 0$

## Graphing Methods

1. Plot points (If it isn't linear, find a lot of points!)

2. Find x intercepts (plug in 0 for y) and y intercepts (plug in 0 for x)

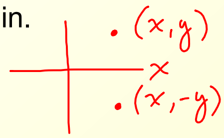
   ex: $y^2 - 4 = x$

   $(-4, 0)$

   $(0, 2)$ $(0, -2)$

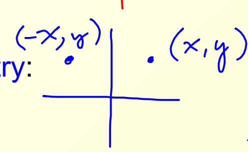
3. Determine if there is symmetry with respect to either axis, or the origin.

x axis symmetry:



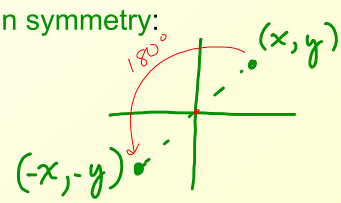

eq → replace y with (-y) and see if it matches original equation

y axis symmetry:




eq → replace x with (-x) & see if it matches

origin symmetry:




eq → replace x & y with (-x) & (-y) and see if it matches.

Does $y^2 - 4 = x$ have x axis, y axis, or origin symmetry?

**x axis symmetry**

$(-y)^2 - 4 = x$

$y^2 - 4 = x$

Matches original!

Has x-axis sym.

**y axis symmetry**

$y^2 - 4 = -x$

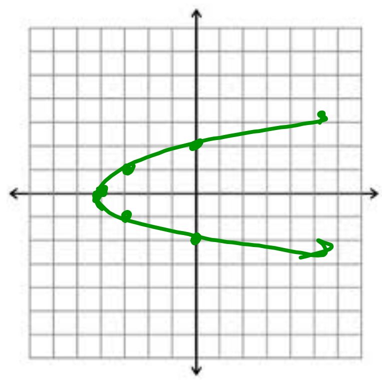
Doesn't match.

**origin symmetry**

$(-y)^2 - 4 = -x$

$y^2 - 4 = -x$

Doesn't match.



How does this help us graph it?

Pick points, use sym. to quickly plot another.

| $x$ | $-4$ | $-3$ | $0$ |
|---|---|---|---|
| $y$ | $0$ | | $\pm 2$ |

Same $x$ will have $\pm y$

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Does $y = x^3 - 4x$ have x axis, y axis, or origin symmetry?

Graph it using that information and the intercepts.

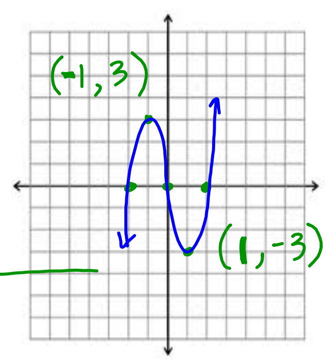
x intercept?

$0 = x(x+2)(x-2)$

$x = 0, \pm 2$

y intercept?

$(0,0)$




x axis sym?

$-y = x^3 - 4x$

y axis sym?

$y = (-x)^3 - 4(-x)$

$y = -x^3 + 4x$

| $x$ | $1$ |
|---|---|
| $y$ | $-3$ |

origin sym?

$-y = (-x)^3 - 4(-x)$

$(-y = -x^3 + 4x)(-1)$

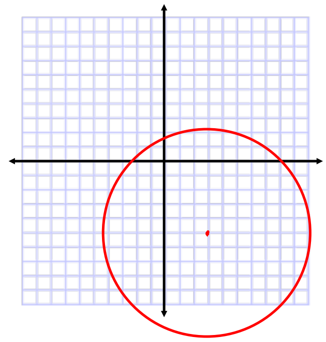
$y = x^3 - 4x$

A circle with center (h,k) and radius r has the equation

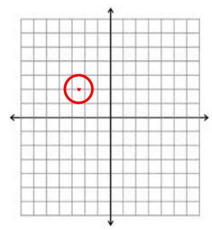
$(x - h)^2 + (y - k)^2 = r^2$

Sketch:

a. $(x - 3)^2 + (y + 5)^2 = 50$



b. $4x^2 + 4y^2 + 20x - 16y + 37 = 0$



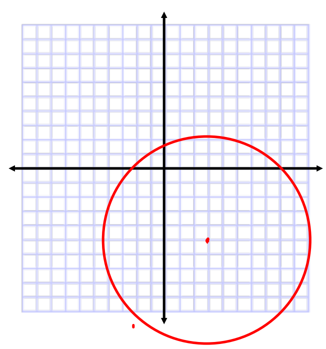
A circle with center (h,k) and radius r has the equation

$(x - h)^2 + (y - k)^2 = r^2$

Sketch:

a. $(x - 3)^2 + (y + 5)^2 = 50$

center: $(3,-5)$ $r = 5\sqrt{2}$
$\approx 7.1$

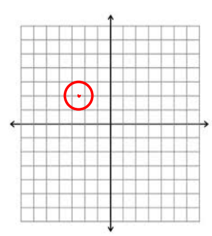


b. $4x^2 + 4y^2 + 20x - 16y + 37 = 0$

$4(x^2 + 5x + \frac{25}{4}) + 4(y^2 - 4y + 4) = -37 + 25 + 16$

$4(x+\frac{5}{2})^2 + 4(y-2)^2 = 4$

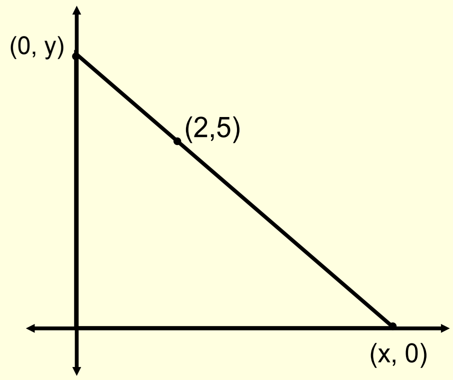
$(x+\frac{5}{2})^2 + (y-2)^2 = 1$



## Explore:

A right triangle is formed in the first quadrant by the x and y axis and a line through (2,5). Write the area of the triangle as a function of x. (This means: find a formula for area in terms of x only.) State the domain of this function.

$A = \frac{1}{2} x y$




Find area when x = 8.

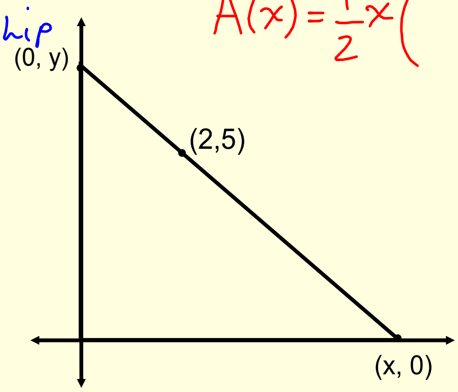
## Group Explore:

A right triangle is formed in the first quadrant by the x and y axis and a line through (2,5). Write the area of the triangle as a function of x. (This means: find a formula for area in terms of x only.) State the domain of this function.

$A(x) = \frac{1}{2} x \left( \quad \right)$

We need a 2nd relationship in x & y... use slope:

$\frac{y-5}{0-2} = \frac{5-0}{2-x}$

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## Group Explore:

A right triangle is formed in the first quadrant by the x and y axis and a line through (2,5). Write the area of the triangle as a function of x. (This means: find a formula for area in terms of x only.) State the domain of this function.

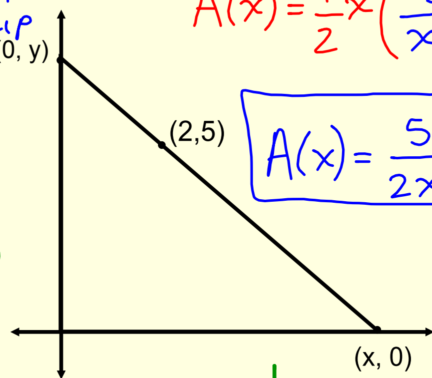
We need a 2nd relationship in x & y... use slope:

$$\frac{y-5}{0-2} = \frac{5-0}{2-x}$$

$$y = \frac{-10}{2-x} + 5\left(\frac{2-x}{2-x}\right)$$

$$y = \frac{-10+10-5x}{2-x}$$

$$y = \frac{5x}{x-2}$$




$$A(x) = \frac{1}{2}x\left(\frac{5x}{x-2}\right)$$

$$\boxed{A(x) = \frac{5x^2}{2x-4}}$$

Find area when x = 8. → plug in 8!

A relation is simply a relationship between variables. A FUNCTION is a relation where each element, x, in the domain has one element, y, in the range.

domain = x = input = independent variable
range = y = output = dependent variable

For the following relations, is y a function of x?

a. (1,2) (2,3) (3,2)

Yes, an input of 1 has only one output (2)
an input of 2 has only one output (3)
an input of 3 has only one output (2)

b. (1,2) (2,3) (1,4)

No, the input 1 has two different outputs (2 and 4)

Which equation(s) determines y as a function of x?

1. $x^2 + y = 1$

$y = 1 - x^2$

2. $x + 4y^2 = 1$

$\frac{4y^2}{4} = \frac{1-x}{4}$

$\sqrt{y^2} = \pm\sqrt{\frac{1-x}{4}}$

A function: any input for x will have only one output.

Not a function: an input could have two different outputs.
Ex: (-3, 1)

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If y is a function of x, you can use function notation.

For: $y = x^2 - 5$ function notation: $f(x) = x^2 - 5$

What is the domain?

$x = \mathbb{R}$

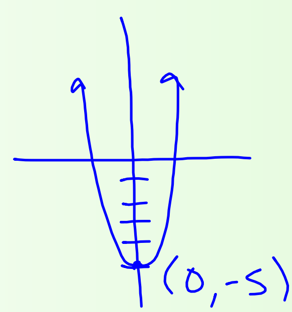
What is the range?

$y \geq -5$

find $f(-3) = (-3)^2 - 5$

find $f(x+3) = (x+3)^2 - 5$

input



$$f(x)=\begin{cases} x^2+1, \underbrace{x<0} \\ x-1, \underline{x\geq 0} \end{cases}$$

Explicit domain for this piece

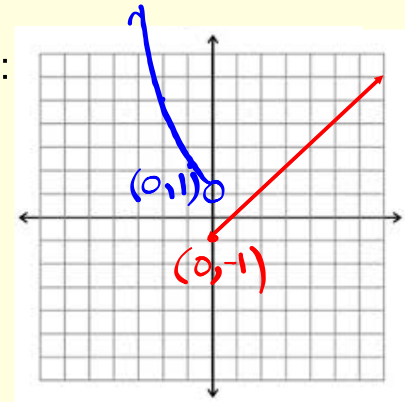
f(x) is called a "piecewise" graph.
Find f(3) and f(-2)

look for the output at $x=3$

or plug 3 into the bottom piece

$f(3)=3-1$

Graph:




Find domain and range of the following functions:

1. $h(x)=\sqrt{4-x^2}$ 2. $g(x)=\dfrac{3}{x-5}$ 3. $A=\pi r^2$

Find domain and range of the following functions

1. $h(x)=\sqrt{4-x^2}$

dom?

$4-x^2 \geq 0$

$x^2-4 \leq 0$

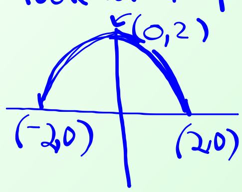
$x^2 \leq 4$

$-2 \leq x \leq 2$

dom: $[-2,2]$

range:

look at graph!




range: $[0,2]$

2. $g(x)=\dfrac{3}{x-5}$

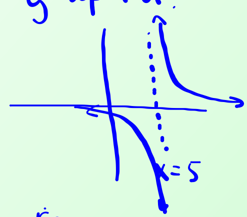
dom?

$x-5 \neq 0$

$x \neq 5$

range?

graph it!




range: $y \neq 0$

3. Area of circle= $\pi r^2$

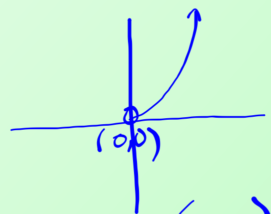
dom?

all reals, but since the radius must be more Than 0,

$(0,\infty)$

range?

graph it!




range: $(0,\infty)$

HW Quiz tomorrow:

SL book p. 57

PC book

pgs. 26, 39, 49, 84, 94

HW: PC book

p. 106 #5, 7, 13, 21, 43,
49, 61, 67

p. 129 # 5 - 13□ (part a & c only),
15, and 25 - 47□

PC book tomorrow and
SL book Wed.